Unit - 1 TESTING OF HYPOTHESIS.

2 marks:

1. What are the expected frequencies of 2x 2 contingency table?

Soln:

$$E(a) = \frac{(a+b)(a+c)}{N} \qquad E(b) = \frac{(a+b)(b+d)}{N}$$

$$E(c) = \frac{(a+c)(c+d)}{N} \qquad E(d) = \frac{(b+d)(c+d)}{N}$$

2. Write down the formula of test stastic to test the significance of difference between the means of large samples.

Soln:

$$Z = \frac{x_1 - x_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

3. What is random sampling?

Suppose we take a sample of size n from a finite population of size N. Then there are N_{C_n} possible samples. A sampling techniques in which each of the N_{C_n} samples has an equal chance of being selected is known as

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random sampling and the sample obtained by this technique is known as random sample.

4. Write about F-test.

If X and Y are two independent chi-square variates with V_1 and V_2 degrees of freedom respectively, F-statistic is given by $F = \dfrac{X/V_1}{Y/V_2}$

5. What are type-I and type II errors? or Define type-I error and type II error in the sampling distribution.

In a statistical hypothesis testing, a type I error is the incorrect rejection of a true null hypothesis (a false positive) while a type II error is incorrectly retaining a false null hypothesis (a false negative)

6. Give the formula for χ^2- test of independence for

a	b
c	d

$$\chi^2 = \frac{N(ad-bc)^2}{(a+b)(a+c)(b+d)(c+d)}$$ where $N = a+b+c+d$

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7. Give the main use of χ^2 test.

χ^2 - distribution is used to test the goodness of fit. It is used to test the independence of attributes

8. A standard sample of 200 tins of coconut oil gave an average weight of 4.95 kgs with a standard deviation of 0.21 kg. Do we accept that the net weight is 5 kgs per tin at 5% significance?

$n = 200 \quad x = 4.95 \quad \sigma = 0.21 \quad \mu = 5$

Null hypothesis : $H_0 : \mu = 5$

Alternative hypothesis : $H_0 : \mu \neq 5$

Test statistic : $z = \dfrac{x - \mu}{\sigma/\sqrt{n}} = -3.367$

$|z| = 3.367$

Tabulated value of z at 5% level of significance is 1.96

Calculated Value > Tabulated Value

H_0 is rejected

We cannot accept that the net weight is 5 kgs per tin.

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9. What is meant by level of significance and critical region?

A region in the sample space S which amount to rejection of H_0 is called as critical region or region of rejection. If ω is the critical region and $t = t(x_1, x_2, \dots x_n)$ is the value of statistic based on a random sample of size n, then $P(t \in \omega | H_0) = \alpha$

$P(t \in \bar{\omega} | H_1) = \beta$.

10. State any two applications of chi-square test

To test the goodness of fit

To test the independence of attributes

To test the homogenity of independent estimates of the population variance.

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Unit - II

DESIGN OF EXPERIMENT

1. What do you understand by design of an experiment?

The design of experiment is designed as as the logical construction of the experiment in which the degree of uncertainty with which the inference is drawn ~~by~~ may be well-defined.

2. What are the basic principles of the design of experiments? or State the principles of Design of Experiments.

→ Replication
→ Randomisation
→ Local control.

3. Write two advantages of completely randomized experimental design?

→ It is easy to lay out the design
→ It allows ~~for~~ for complete flexibility.

Any number of factor classes and replications may be used.

4. Is a 2x2 Latin square design possible? why?

No. For $n=2$, d.f of SSE $=0$ and hence MSE is not defined. Comparisons are not possible. Hence a 2x2 Latin square design is not possible

5. what is ANOVA?

The technique of analysis of variance is referred to as ANOVA. A table showing the source of variation, the sum of squares, the degree of freedom, mean square (Variance) and the formula for F-ratio is known as ANOVA table.

6. Define experimental error.

The estimation of the amount of variation due to each of the independent factors seperately and then comparing these estimates due to assignable factors with the estimate due to the chance factor is known as experimental error or simple error.

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7. What are the uses of ANOVA?

→ To test the homogeneity of several means

→ The ANOVA technique is now frequently applied in testing the linearity of the fitted regression line or the significance of the correlation ratio.

8. What is the aim of the design of experiment.

The main aim of the design of experiment is to control the extraneous variables and hence to minimise the experimental error so that the results of the experiment could be attributed only to the experimental variable.

9. What is a completely randomized design?

Let us consider K treatments in an experiment. Let the treatment be replicated r_i times $(i = 1, 2, \ldots K)$. The total number of experimental units required for the design may be expressed $\sum_{i=1}^{k} r_i = N$. Here the different treatments to be applied is completely random, so that any material to which the treatments might be applied is considered to be approximately homogeneous.

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Unit - III.

SOLUTION OF EQUATION AND EIGEN VALUE PROBLEMS.

1. State the Newton - Raphson method formula and the criteria for convergence.

Newton - Raphson method formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Criteria for convergence : $|f(x) f''(x)| < |f'(x)|^2$

2. Mention the order and condition for the convergence of Newton - Raphson method.

The order of convergence is 2.

Condition for convergence is $|f(x).f''(x)| < |f'(x)|^2$

3. Obtain the iterative formula to find $1/N$ using Newton-Raphson method. Or Find the iterative formula by Newton's method for $1/N$ where N is a positive integer. Or Establish an iteration formula to find the reciprocal of a positive number N by Newton - Raphson

method.

$$x = 1/N$$

$$N = 1/x$$

$$1/x - N = 0$$

$$f(x) = 1/x - N$$

$$f'(x) = -1/x^2$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n}{-1/x_n^2}$$

$$= x_n + x_n^2 (1/x_n - N)$$

$$x_{n+1} = x_n [2 - N x_n]$$

4. Derive a formula to find the Value of $N^{1/2}$ and $1/N$ where $N \neq 0$. Using Newton method.

1) Let $x = \sqrt{N}$

$$x^2 = N$$

$$x^2 - N = 0$$

$$f(x) = x^2 - N$$

$$f'(x) = 2x$$

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$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n^2 - N}{2x_n}$$

$$= \frac{2x_n^2 - x_n^2 + N}{2x_n}$$

$$= \frac{x_n^2 + N}{2x_n}$$

$$x_{n+1} = \frac{1}{2}\left[x_n + \frac{N}{x_n}\right]$$

ii) $x = 1/N$

$N = 1/x$

$1/x - N = 0$

$f(x) = 1/x - N$

$f'(x) = -1/x^2$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{1/x_n - N}{-1/x^2}$$

$$= x_n + x_n^2 (1/x_n - N)$$

$$x_{n+1} = x_n [2 - N x_n].$$

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5. What are the merits of Newton-Raphson method?

Soln:

1. Newton's method is successfully used to improve the result obtained by other methods.

It is applicable to the solution of equation involving algebraical functions as well as transcendental functions.

6. Find the smallest positive root of $x^3 - 2x + 0.5 = 0$

Soln: $f(x) = x^3 - 2x + 0.5$

$f'(x) = 3x^2 - 2$

$f(0) = 0.5 (+ve)$

$f(1) = -0.5 (-ve)$

Root lies between 0 and 1

$|f(0)| < |f(1)|$

$\therefore$ The root is nearer to 0

Let $x_0 = 0$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.25$$

$$x_2 = x_1 - \frac{f(x_0)}{f'(x_0)} = 0.25 - (0.0152 / -1.8125)$$

$$= 0.2586$$

7. Perform four iterations of the Newton Raphson method to find the smallest positive root of the equation $f(x) = x^3 - 5x + 1 = 0$.

Soln: $f(x) = x^3 - 5x + 1 = 0$

$f'(x) = 3x^2 - 5$

$f(0) = 1$ (+ve)

$f(1) = -3$ (-ve)

Root lies between 0 and 1

$|f(0)| < |f(1)|$

$\therefore$ The root is nearer to 0

Let $x_0 = 0.5$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.5 - 0.3235 = 0.1765$$

$x_2 = 0.2016$

$x_3 = 0.2016$

$x_4 = 0.2016$

8. What is the procedure of Gauss Jordan method?

Soln: Co-efficient matrix is transformed into diagonal matrix

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9. Solve the equations $5x-2y=1$, $4x+28y=23$ using the Gauss elimination method.

Soln:

$$[A, B] = \begin{bmatrix} 5 & -2 & 1 \\ 4 & 28 & 23 \end{bmatrix}$$

$$\sim \begin{bmatrix} 5 & -2 & 1 \\ 0 & 148 & 111 \end{bmatrix} \quad R_2 \to 5R_2 - 4R_1$$

By back substitution,

$$148y = 111$$

$$y = 0.75$$

$$5x - 2y = 1$$

$$5x = 2.5$$

$$x = 0.5$$

---

10. Solve by Gauss elimination method $10x + y = 18.141$; $x + 10y = 28.140$.

Soln:

$$[A, B] = \begin{bmatrix} 10 & 1 & 18.141 \\ 1 & 10 & 28.140 \end{bmatrix}$$

$$\sim \begin{bmatrix} 10 & 1 & 18.141 \\ 0 & 99 & 26.259 \end{bmatrix} \quad R_2 \to 10R_2 - R_1$$

By back substitution,

$$99y = 263.259$$

$$y = 2.66$$

$$10x + y = 18.141$$

$$x = 1.5481$$

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11. Distinguish between Gauss elimination and Gauss Jordan methods.

Soln:

Gauss elimination method	Gauss Jordan method
Co-efficient matrix is transformed into upper triangular matrix	Co-efficient matrix is transformed into diagonal matrix
Direct method	Direct method
we obtain the solution by back substitution method.	No need to back substitution method.
It is performed for large system of equations	It is performed for smaller system of equations.
Less number of multiplication are required compared to Gauss Jordan method.	More number of multiplication are required compared to Gauss - elimination method.

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12. Write a sufficient condition for Gauss-Seidal method to Converge.

Soln:

In each equation of the system, the absolute value of the largest co-efficient is greater than the sum of the absolute value of all the remaining co-efficient which is called diagonally dominant.

13. Compare Gauss elimination with Gauss Seidal

Soln:

Gauss elimination method	Gauss Seidal method
It works on the basis of elimination of variables	It employs initial values and iterates to obtain refined estimates.
Direct method These can be used for small number of Co-efficient	Indirect method. There is no round-off error problem. Error is controlled by number of iterations.
Its performance is affected due to round - off errors.	These can be used for large number of co-efficients.

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It gives the exact solution in finite number of steps	successive approximations get the solution

14. What kind of an eigenvalue and eigenvector of a matrix would be obtained by power method?

Soln:

The dominant eigenvalue and the corresponding eigenvector can be obtained by power method.

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UNIT-IV.

Interpolation, Numerical Differentiation and Integration.

1. Gn $f(2)=5$, $f(2.5)=5.5$ find the linear interpolating polynomial Lagrange interpolation.

Soln:

$x_0=2$, $x_1=2.5$, $y_0=5$, $y_1=5.5$

$$y=f(x)=\frac{(x-x_1)}{(x_0-x_1)}y_0+\frac{(x-x_0)}{(x_1-x_0)}y_1$$

$$=\frac{(x-2.5)}{(-0.5)}(5)+\frac{(x-2)}{(0.5)}(5.5)$$

$$=(x-2.5)(-10)+(x-2)(11)$$

$$=-10x+25+11x-22$$

$$=x+3.$$

2. Construct the divided difference table for data

x	0.5	1.5	3.0	5.0	6.5	8
$f(x)$	1.625	5.875	31.0	131	285.125	521

Soln:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
0.5	1.625					
		4.25				
1.5	5.875		7			
		24.125		1		
					0	
3	31		11			
		50				
5	131					

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x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
			16	1		
5	131	118				
6	281.125		22		0	0
		228		1		
8	521					

3. Give the Newton's backward difference table for

x :	0	1	2	3
y :	-1	-2	-1	2

Soln:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	-1			
		-1		
1	-2		2	
		1		0
2	-1		2	
		3		
3	2			

4. Compare Trapezoidal rule with Simpson's 1/3 rule

Trapezoidal	Simpson's 1/3 rule.
It has no specific restriction on number of segments.	It requires even number of segments and odd number of points

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Degree of $y(x)$ is one	Degree of $y(x)$ is two
$\lvert E\rvert < \frac{(b-a)h^2}{12} M$	$\lvert E\rvert < \frac{(b-a)h^4}{180} M$
Order h^2	Order h^4
It is approximated by trapezoids	It is approximated by set of parabolas.

5. Write down the errors in Trapezoidal and Simpson's rules of numerical integration

The error in Trapezoidal rule is

$$\lvert E\rvert < \frac{(b-a)h^2}{12} M$$

The error in Simpson's 1/3 rule is

$$\lvert E\rvert < \frac{(b-a)h^4}{180} M$$

6. Specify the Newton's backward difference formulae for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

Soln:

$$\frac{dy}{dx} = \frac{1}{h}\left[\nabla y_n + \left(\frac{2v+1}{2}\right)\nabla^2 y_n + \left(\frac{3v^2+6v+2}{6}\right)\nabla^3 y_n + \left(\frac{4v^3+18v^2+22v+6}{24}\right)\nabla^4 y_n + \cdots\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\nabla^2 y_n + (v+1)\nabla^3 y_n + \left(\frac{6v^2+18v+11}{12}\right)\nabla^4 y_n + \cdots\right]$$

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7. Find the polynomial which takes the following values $f(0)=-1$, $f(1)=1$ and $f(2)=4$ using the Newton's interpolating formula.

Soln:

$$y(x)=y_0+\frac{u}{1!}\Delta y_0+\frac{u(u-1)}{2!}\Delta^2 y_0$$

x	y	Δy	$\Delta^2 y$
0	-1		
		2	
1	1		1
		3	
2	4		

$$u=\frac{x-x_0}{h}=x$$

$$y(x)=-1+2x+\frac{x^2-x}{2}$$

$$y(x)=-1+\tfrac{3}{2}x+\tfrac{x^2}{2}$$

8. State two properties of divided differences

* The divided difference operator is linear
* The n^{th} order divided difference polynomial of degree n are constant, equal to the co-efficient of x^n

9. Apply Newton's backward formula to find a polynomial of degree 3.

x	3	4	5	6
y	6	24	60	120

Soln:

$$y(x)=y_n+v\nabla y_n+\frac{v(v+1)}{2!}\nabla^2 y_n+\frac{v(v+1)(v+2)}{3!}\nabla^3 y_n$$

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x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
3	6			
		18		
4	24		18	
		36		6
5	60		24	
		60		
6	120			

$$y(x) = y_3 + v\nabla y_3 + \frac{v(v+1)}{2!}\nabla^2 y_3 + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_3$$

$$y(x) = 120 + 60v + \frac{v(v+1)}{2!}24 + \frac{v(v+1)(v+2)}{3!}6$$

$$v = \frac{x - x_n}{h} = x - 6$$

$$y(x) = 124 + 78(x-6) + 14(x-6)^2.$$

10 What is meant by interpolation?

The process of finding the value of a function inside the given range is called interpolation.

11. What is the order of error in trapezoidal and Simpson's one-third rules?

The order of error in trapezoidal rule is of order h^2.

The order of error in trapezoidal rule is of order h^4.

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UNIT-V.

Numerical Solution of ordinary differential eqn.

1. Write down the Euler formula for

$y' = f(x,y)$, $y(x_0) = y_0$

Soln:

$$y_{n+1} = y_n + hf(x_n, y_n), n = 0, 1, 2, \ldots$$

2. Gn $y' = \dfrac{y-x}{x+y}$ with initial condition $y=1$ at $x=0$ find y for $x=0.1$ by Euler's method.

$f(x,y) = \dfrac{y-x}{x+y}$, $x_0 = 0$, $y_0 = 1$, $h = 0.1$, $x_1 = 0.1$, $y_1 = ?$

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y(0.1) = y_1 = y_0 + (0.1) f(x_0, y_0)$$
$$= 1 + (0.1)(1)$$
$$= 1.1$$

3. If $y' = -y$, $y(0) = 1$ then find $y(1)$ by Euler method

$f(x,y) = -y$

$y_{n+1} = y_n + hf(x_n, y_n)$ $x_0 = 0$, $y_0 = 1$, $h = 0.1$, $x_1 = 0.1$
$y_1 = ?$

$$y(0.1) = y_1 = y_0 + (0.1) f(x_0, y_0)$$
$$= 1 + (0.1)(-1)$$
$$= 1 - 0.1$$
$$= 0.9$$

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4. Find $y(0.1)$ by Euler's method, if $\frac{dy}{dx} = x^2+y^2$, $y(0)=0.1$

$f(x,y) = x^2+y^2$, $x_0=0$, $y_0=0.1$, $h=0.1$, $x_1=0.1$, $y_1=?$

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y(0.1) = y_1 = y_0 + (0.1) f(x_0, y_0)$$

$$= 0.1 + (0.1)(0.01)$$

$$= 0.1 + 0.001$$

$$= 0.101$$

---

5. State the modified Euler's formula for first order ordinary differential equation. Or write down the modified Euler formulae for $y' = f(x,y)$

$$y_{n+1} = y_n + h\left[f\left(x_n + \tfrac{1}{2}h, y_n + \tfrac{1}{2}hf(x_n, y_n)\right)\right] \quad n=0,1,2,\ldots$$

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6. Given the initial value problem $u' = -2tu^2$, $u(0)=1$ estimate $u(0.4)$ using modified Euler Cauchy method.

$$u_{n+1} = u_n + h\left[f\left(t_n + \tfrac{1}{2}h, u_n + \tfrac{1}{2}hf(t_n, u_n)\right)\right].$$

$$u_1 = u_0 + h\left[f\left(t_0 + \tfrac{1}{2}h, u_0 + \tfrac{1}{2}hf(t_0, u_0)\right)\right].$$

$f(u,t) = -2tu^2$ $t_0=0$, $u_0=1$, $h=0.4$, $t_1=0.4$, $u_1=?$

$$u_1 = 1 + (0.4)\left[f(0.2, 1)\right]$$

$$= 1 + (0.4)(-0.4)$$

$$= 1 - 0.16$$

$$= 0.84$$

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7. Using Taylor's method find y at $x=0.1$ when $y = x^2 - y$, $y(0)=1$.

$x_0=0,\ y_0=1,\ h=0.1,\ x_1=0.1,\ y_1=?$

$y' = x^2 - y$, $y_0' = x_0^2 - y_0 = -1$

$y'' = 2x - y'$, $y_0'' = 2x_0 - y_0' = 1$

$y''' = x - y''$, $y_0''' = 2 - y_0'' = 1$

$y^{IV} = -y'''$, $y_0^{IV} = -y_0''' = -1$

By Taylor's series,

$$y_1 = y_0 + hy_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + \frac{h^4}{4!}y_0^{IV} \ldots$$

$$y_1 = 1 + (0.1)(-1) + \frac{(0.1)^2}{2!}(1) + \frac{(0.1)^3}{3!}(1) + \frac{(0.1)^4}{4!}(-1) + \ldots$$

$= 1 - 0.1 + 0.005 + 0.00016 - 0.0000042$

$= 0.90516$

8. Compute $y(0.1)$ correct to 4 decimal places if $y(x)$ satisfies $y' = x + y$, $y(0)=1$ by Taylor's series method.

$x_0=0,\ y_0=1,\ h=0.1,\ x_1=0.1,\ y_1=?$

$y' = x + y$, $y_0' = x_0 + y_0 = 1$

$y'' = 1 + y'$, $y_0'' = 1 + y_0' = 2$

$y''' = y''$, $y_0''' = y_0'' = 2$.

By Taylor Series,

$$y_1 = y_0 + hy_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + \frac{h^4}{4!}y_0^{IV} + \ldots$$

$$= 1 + (0.1)(1) + \frac{(0.1)^2}{2!}(2) + \frac{(0.1)^3}{3!}(2) + \ldots$$

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$= 1 + 0.1 + 0.01 + 0.0003$

$= 1.1103$

9. What are single step and multistep methods? Give an example?

Single step method: Euler's method, Taylor's method, R-K method.

ex: $y' = x + y$, $y(0) = 1$

Multistep method: Milne's predictor and corrector method.

Adams-bashforth predictor and corrector method.

ex: $\frac{dy}{dx} = x^3 + y$, $y(0) = 2$, $y(0.2) = 2.073$, $y(0.4) = 2.452$

$y(0.6) = 3.023$

10. What is main difference between single and multistep methods in solving first order ordinary differential equations?

In single step method, the value of y_n is calculated from the value of y_{n-1}. In multi step method, the value of y_n is calculated from the value of y_{n-1}, y_{n-2}, y_0

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